Model order reduction for complex ocular simulations inside the human eyeball

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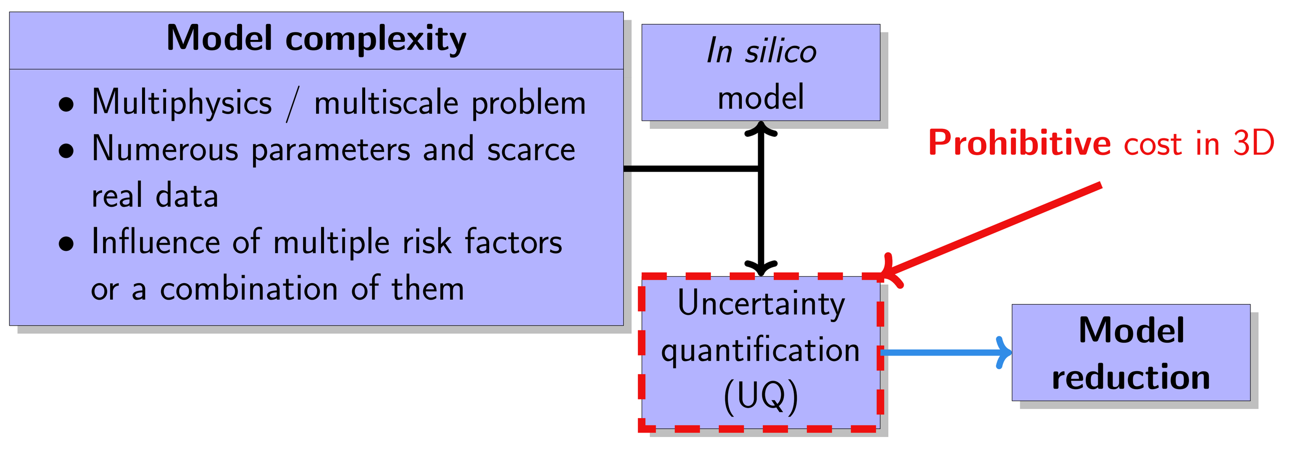
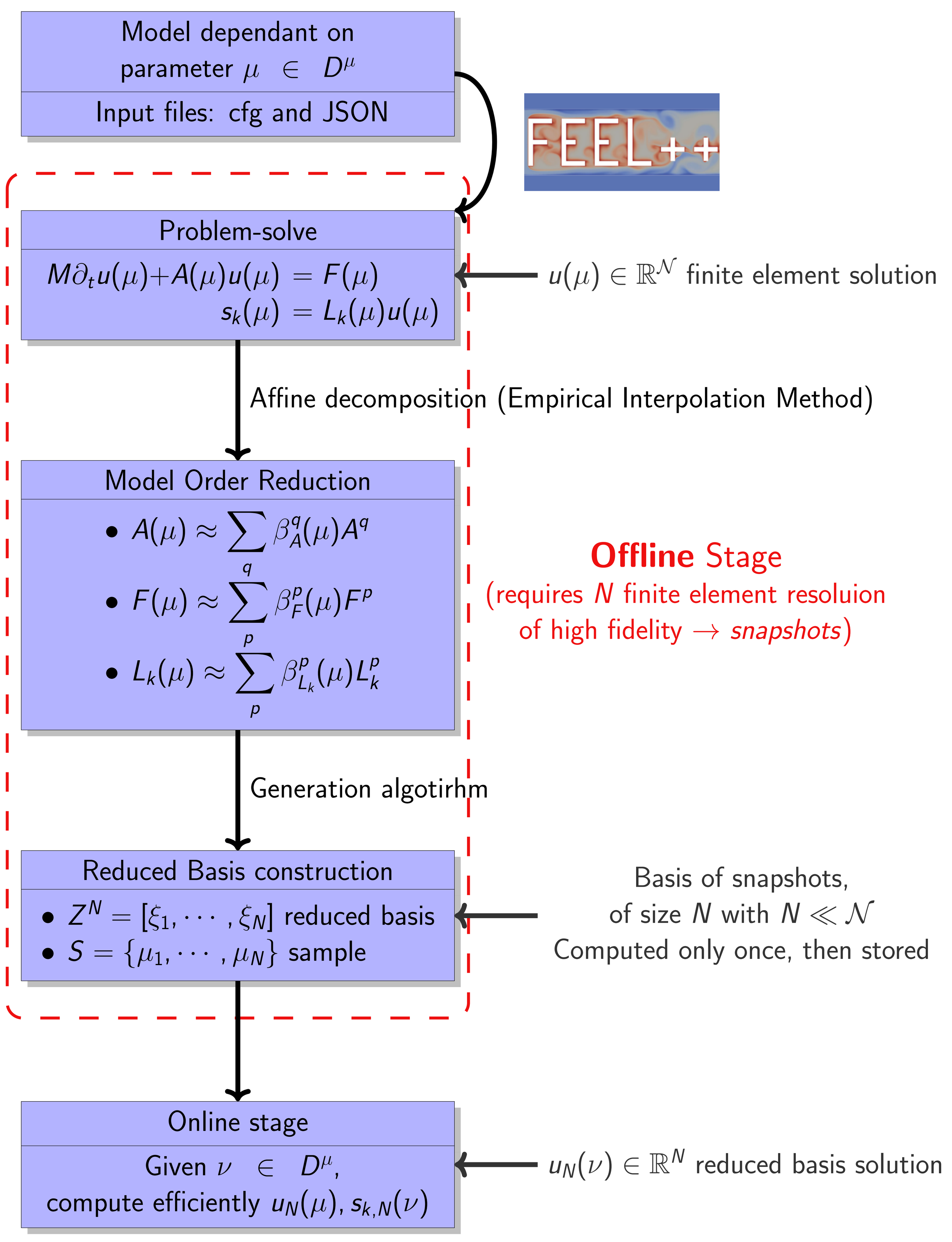
Motivations

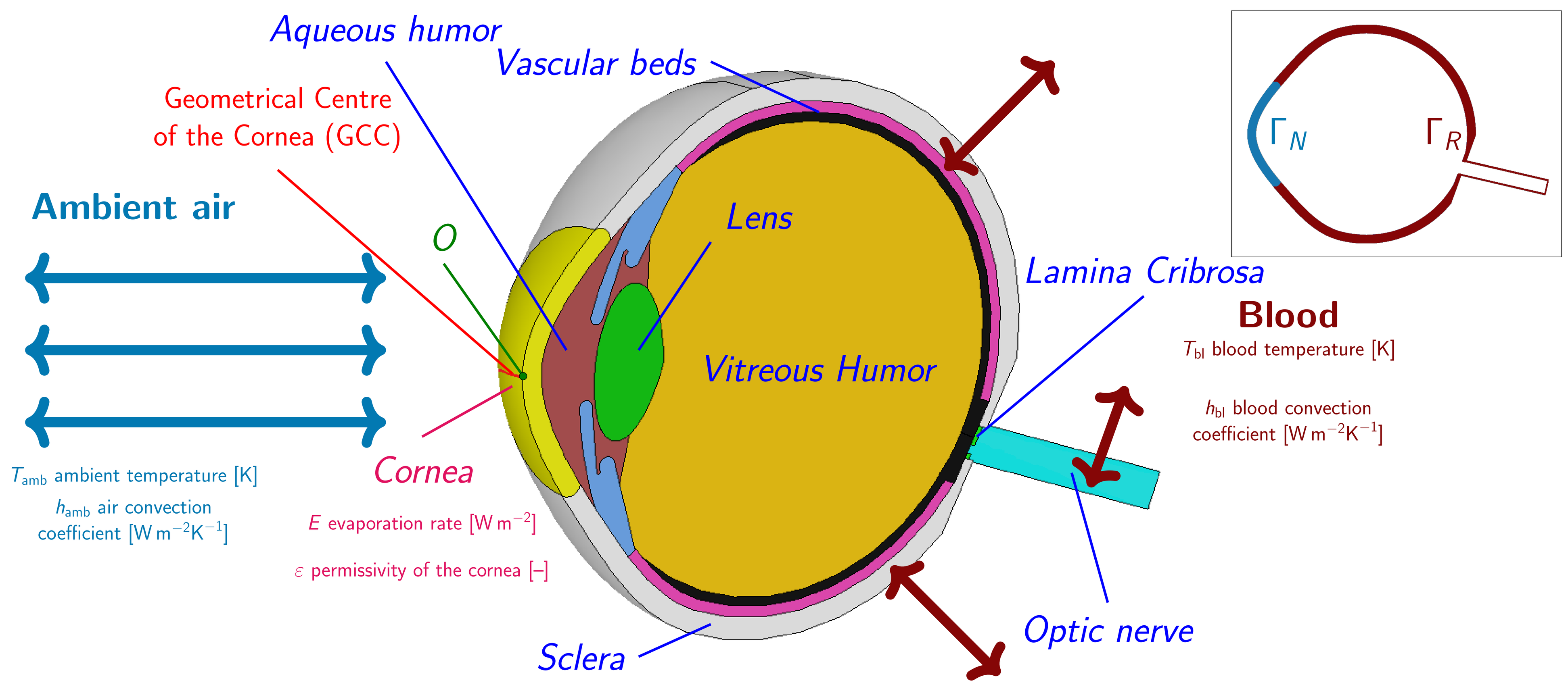
Eye2brain project: develop a reliable and efficient mathematical and computational framework to simulate and predict the functioning and the connection between the eye and the brain



Model Order Reduction with Feel++ [2]



3D parametrized model of the human eye [1]



Heat transfer equation: $\rho_i C_{p,i} \frac{\partial T_i}{\partial t} \nabla \cdot (k_i \nabla T_i) = 0$ on $\Omega = \cup_i \Omega_i$

- $i \in \{1, ..., 10\}$ is the volume index (Cornea, VitreousHumor, Lens, Lamina, OpticNerve...),
- T_i [K] is the temperature in the volume i,
- k_i [W m^{-1}K^{-1}] is the thermal conductivity, ρ_i [kg m^{-3}] is the density and $C_{p,i}$ [J kg^{-1}K^{-1}] is the specific heat.

Boundary conditions:

- Non linear Neumann condition on Γ_N:

$$-k_{\text{cornea}} \frac{\partial T}{\partial \mathbf{n}} = h_{\text{amb}}(T - T_{\text{amb}}) + \sigma\varepsilon(T^4 - T_{\text{amb}}^4) + E$$

  Stefan–Boltzmann constant: $\sigma = 5.670\,\text{W m}^{-2}\text{K}^{-4}$.
  This condition can be approximated as a linear one [4].

- Robin condition on Γ_R: $-k_i \frac{\partial T}{\partial \mathbf{n}} = h_{\text{bl}}(T - T_{\text{bl}})$

Verification and validation of the model

Parameters used in the model (see figure above) :

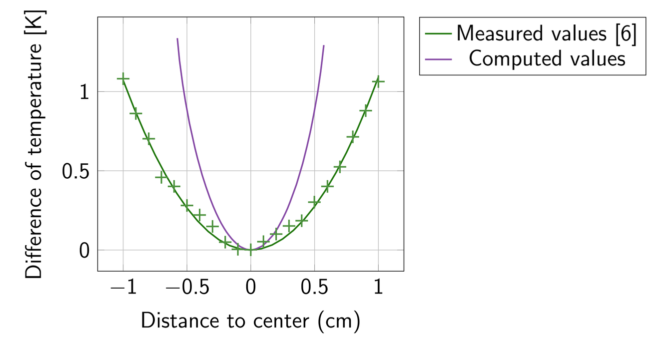
Symbol	Dimension	baseline value
ε	[–]	0.975
T_{amb}	[K]	298
T_{bl}	[K]	310
h_{amb}	[W m^{-2}K^{-1}]	10
h_{bl}	[W m^{-2}K^{-1}]	65
E	[W m^{-2}]	40
k_{lens}	[W m^{-1}K^{-1}]	0.40

Baseline values are taken from [6].

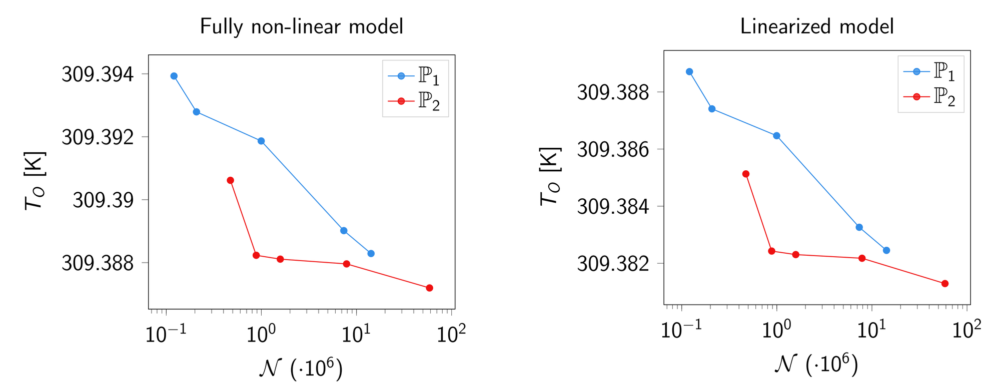
Order of convergence for a toy problem with analytical solution:

Norm	$\mathbb{P}_1$-elements		$\mathbb{P}_2$-elements	
	Expected slope	Observed slope	Expected slope	Observed slope
L^2	2	2.378	3	3.628
H^1	1	1.170	2	2.316

Comparaison with measured data over GCC:



Mesh convergence study:



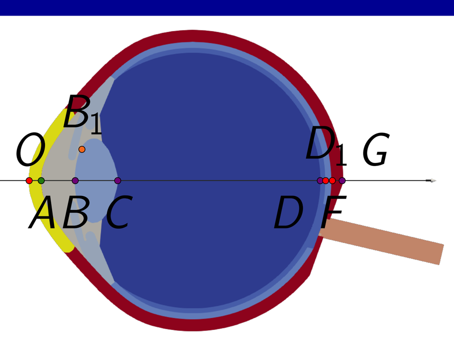
Ocular surface temperature:

Model prediction	307.98 K
Experimental literature [3]	307.80 K

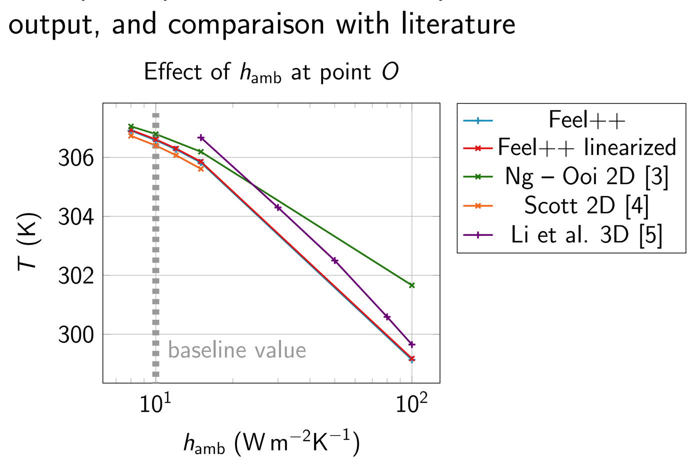
Uncertainty quantification

Deterministic analysis:

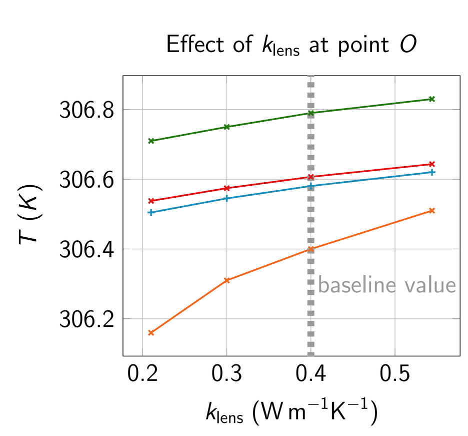
One parameter can vary, while all the other are fixed to the baseline value.
We focus on specific locations and the mean value of the temperature in the cornea.



Featured geometrical locations for the output of interest (temperature)

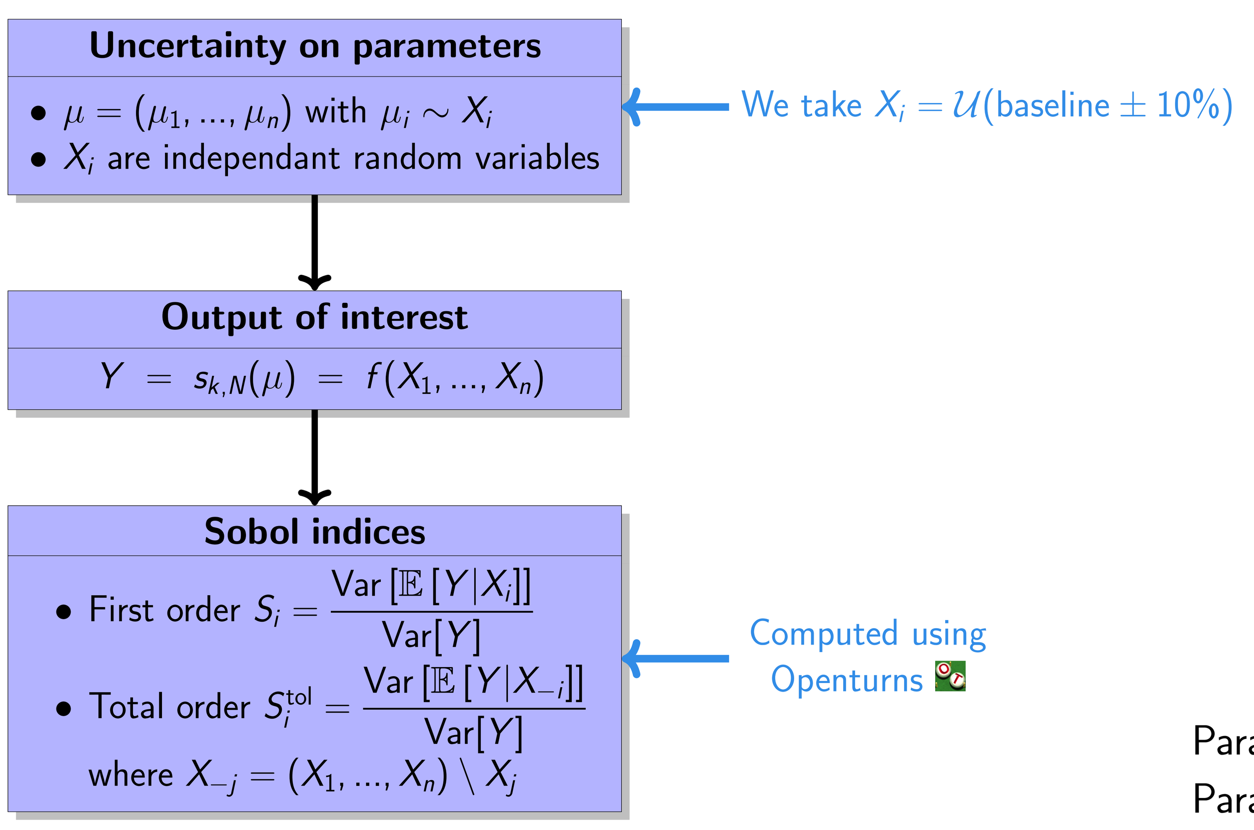


Example of parameter with an impact on the output, and comparison with literature

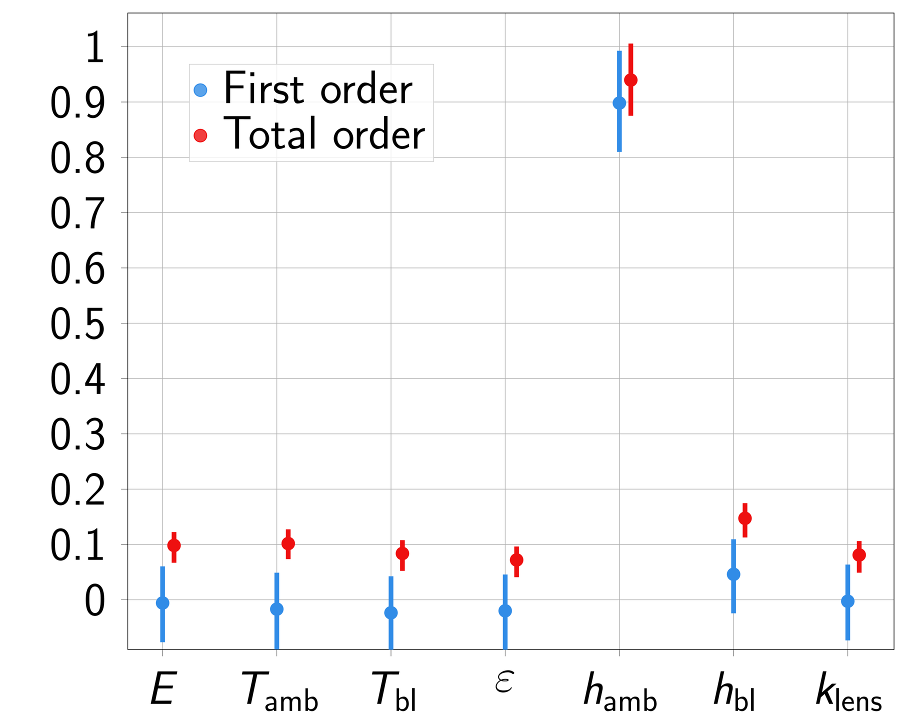


Parameter with no impact on the output

Sensitivity analysis:



Sobol indices on the output T_O:



Parameters with significant impact: h_{amb} then h_{bl}.
Parameters with moderate or minimal impact: ε, T_{bl}...
Total order indices show that interaction of higher order are present.

Conclusion and next steps

- Set up of a complex framework to assess *via* model reduction and sensitivity analysis the influence of parameters on heat transfer in the human eye.
- More complex models: coupling with aqueous humor fluid dynamics, include multiscale aspects (IOP dynamics described by a non linear ODE), assess influence of geometric parameters (such as cornea thickness).
- Potential clinical application: local drug delivery in the eye, influence of aging.

References

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6. N. Efron et al. (1989). "Ocular surface temperature." In: *Current eye research* 8 9, pp. 901–6