A coupled fluid-dynamics-heat transfer model for 3D simulations of the aqueous humor flow in the human eye

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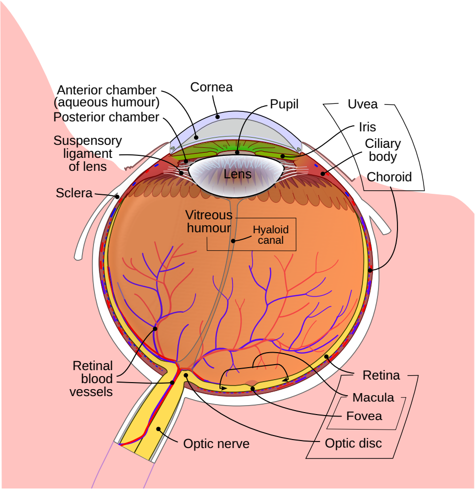
Context



Rhcastilhos, from Wikipedia

- Understanding the human eye involves studying **complex interactions between physical phenomena** such as heat transfer and fluid dynamics.
- **Accurate** computational models are crucial for simulating ocular system physiology and understanding ocular diseases.
- We focus on simulating **aqueous humor** flow and its coupling with **heat transfer** in the eye.

Description of the geometry

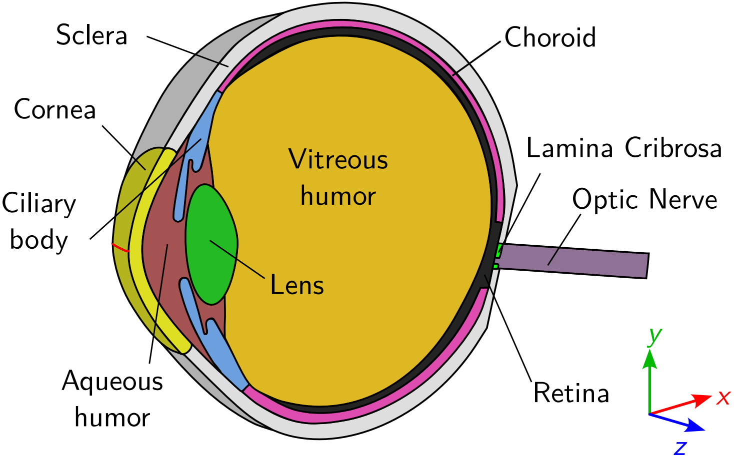


Figure 1: Geometrical model of the human eye.

Aqueous humor (AH)

- Transparent fluid produced by the **ciliary body**, that flows from the posterior chamber to the anterior chamber.
- Plays a fundamental role in the maintenance of the **intraocular pressure** level.
- In addition to the hydraulic pressure difference created by production and drainage, AH dynamic is influenced by **posture and thermal factors**.

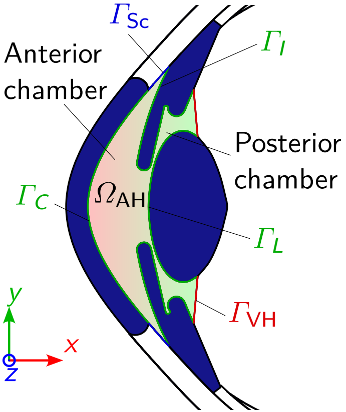


Figure 2: Geometrical model of the human eye.

Biophysical model[1,2]

- Imcompressible fluid, constant density,
- The steady flow of the aqueous humor is governed by the Navier–Stokes equations:

Navier-Stokes equations | Boussinesq approximation

$$\rho(\boldsymbol{u} \cdot \nabla \boldsymbol{u}) - \mu \nabla^2 \boldsymbol{u} + \nabla p = -\rho\beta(T - T_{\mathrm{ref}})\boldsymbol{g} \quad \text{in } \Omega_{\mathrm{AH}},$$

Mass conservation

$$\nabla \cdot \boldsymbol{u} = 0 \quad \text{in } \Omega_{\mathrm{AH}},$$

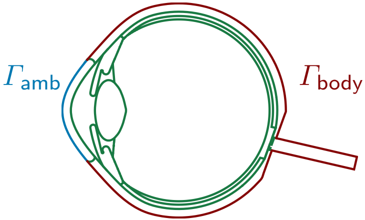
Heat transfer equation

$$\rho C_p \boldsymbol{u} \cdot \nabla T - k \nabla^2 T = 0 \quad \text{in } \Omega,$$

[1] Abdelhafid et al. "Operator Splitting for the Simulation of Aqueous Humor Thermo-Fluid-Dynamics in the Anterior Chamber". In: *Recent Developments in Mathematical, Statistical and Computational Sciences* (2021).

[2] Wang et al. "luid and structure coupling analysis of the interaction between aqueous humor and iris". In: *BioMedical Engineering OnLine* (2016).

Biophysical model



- Conditions on velocity:

$$\boldsymbol{u} = \mathbf{0} \quad \text{on } \Gamma_C \cup \Gamma_I \cup \Gamma_L \cup \Gamma_{\mathrm{VH}} \cup \Gamma_{\mathrm{Sc}},$$

- Thermal exchanges with surrounding body and ambient air:

$$-k\frac{\partial T}{\partial \boldsymbol{n}} = h_{\mathrm{bl}}(T - T_{\mathrm{bl}}) \quad \text{on } \Gamma_{\mathrm{body}},$$
$$-k\frac{\partial T}{\partial \boldsymbol{n}} = (h_{\mathrm{amb}} + h_{\mathrm{r}})(T - T_{\mathrm{amb}}) + E \text{ on } \Gamma_{\mathrm{amb}}.$$

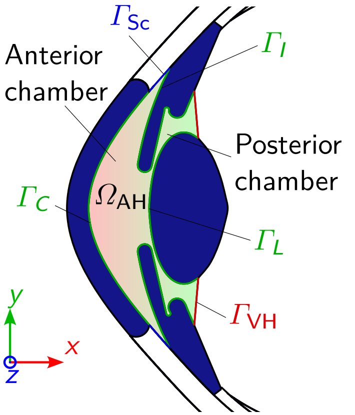
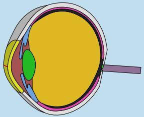


Figure 3: Description of boundary conditions.

Computational framework

Geometrical model



Mesh generation

- 3D mesh with **$4.97 \cdot 10^6$ elements**,
- **Fine mesh refinement** in Ω_{AH}, where the coupled model is considered.

Biophysical model

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$$\nabla \cdot \boldsymbol{u} = 0 \quad \text{in } \Omega_{\text{AH}},$$
$$\rho C_p \boldsymbol{u} \cdot \nabla T - k \nabla^2 T = 0 \quad \text{in } \Omega,$$

Finite element solver

- Use the Feel++[a] **heatfluid** toolbox using monolithic approach and **PDE based preconditioning**,
- Compute solution on a cluster `AMD EPYC 7713 64-Core Processor`, parallelized on 128 cores : **a few minutes**.

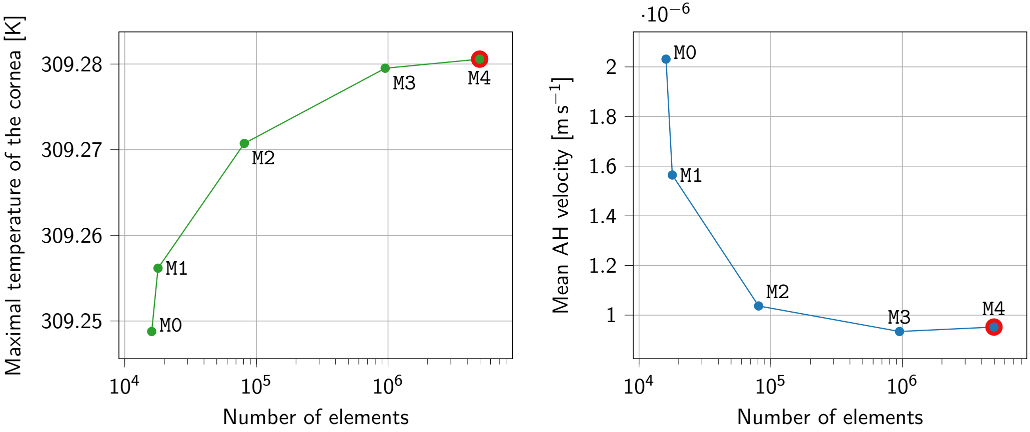
[a] github.com/feelpp/feelpp

Mesh convergence study

M	h_{min}	h_{max}	h_{mean}	# elements	# Degree of freedom		
					T	$\boldsymbol{u}$	p
M0	$1.21 \cdot 10^{-4}$	$7.59 \cdot 10^{-3}$	$2.53 \cdot 10^{-3}$	16,027	2,928	15,105	852
M1	$1.41 \cdot 10^{-4}$	$7.42 \cdot 10^{-3}$	$2.38 \cdot 10^{-3}$	17,899	3,275	17,205	957
M2	$1.15 \cdot 10^{-4}$	$5.75 \cdot 10^{-3}$	$1.07 \cdot 10^{-3}$	80,924	14,213	95,670	4,956
M3	$3.15 \cdot 10^{-5}$	$3.25 \cdot 10^{-3}$	$3.46 \cdot 10^{-4}$	$9.52 \cdot 10^{5}$	$1.64 \cdot 10^{5}$	$1.19 \cdot 10^{6}$	56,120
M4	$2.01 \cdot 10^{-5}$	$3.07 \cdot 10^{-3}$	$1.65 \cdot 10^{-4}$	$4.97 \cdot 10^{6}$	$8.4 \cdot 10^{5}$	$6.79 \cdot 10^{6}$	$3.06 \cdot 10^{5}$

Table 1: Caracteristics of meshes used for the convergence study.

Mesh convergence study



(a) Temperature (b) Fluid velocity

Validation: temperature at the surface of the cornea

Author	T_{amb}	No AH flow	AH flow coupled		
			Prone	Supine	Standing
Scott[3] (2D)	293.15	306.4	–	–	–
Ooi et al.[4] (2D)	298	306.45	–	–	306.9
Karampatzakis et al.[5] (3D)	293	306.81	–	–	307.06
	296	307.33	–	–	307.51
	298	307.69	–	–	307.83
Current model (3D)	293	306.5599	306.5644	306.5542	306.6323
	296	307.1080	307.1114	307.104	307.1559
	298	307.4735	307.4760	307.4704	307.5078

Table 1: Corneal surface temperature for varitous configuration. Temperatures are given in K.

[3] Scott. *Physics in Medicine* (1988).
[4] Ooi et al. *Computers in Biology and Medicine* (2008).
[5] Karampatzakis et al. *Physics in Medicine & Biology* (2010).

Temperature

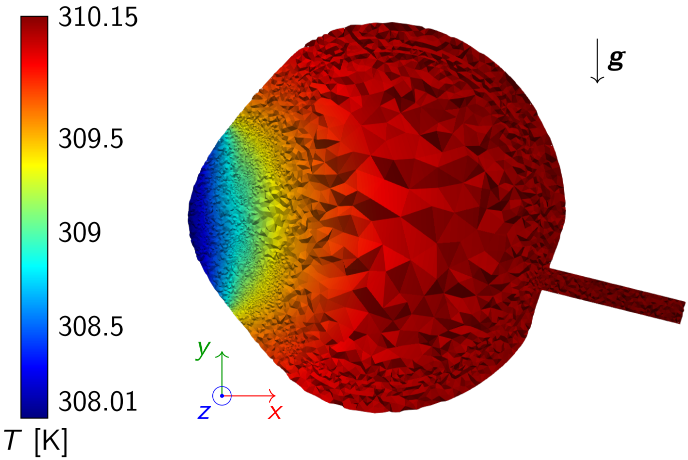


Figure 4: Temperature over the eyeball in the standing position, on a vertical cut, and mesh discretization.

Pressure and velocity: impact of the posture

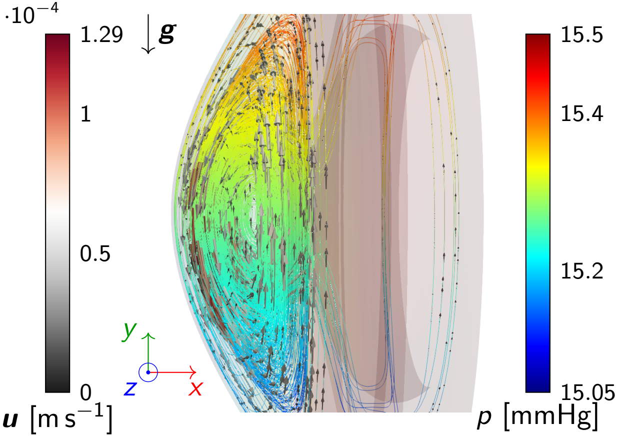


Figure 5: Standing position.

- **Recirculation** of the AH,

[a] Wang et al., *BioMedical Engineering OnLine* (2016).
[b] Abdelhafid et al., *Recent Developments in Math., Statistical and Computational Sciences* (2021).
[c] Murgoitio-Esandi et al., Translational Vision Science & Technology (2023).

Pressure and velocity: impact of the posture

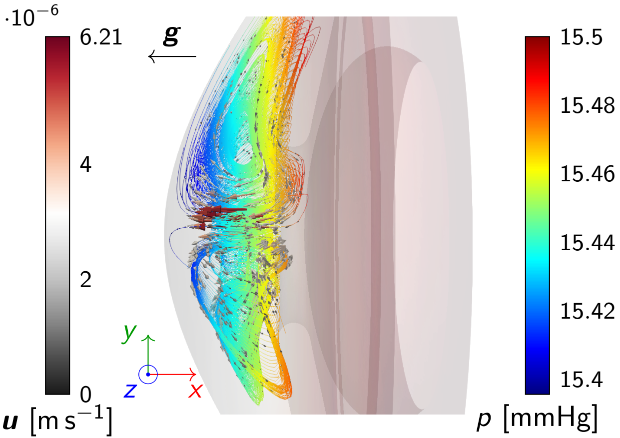


Figure 5: Prone position.

- **Recirculation** of the AH,
- Formation of a **Krukenberg's spindle**, in good agreement with clinical observations and previous studies[a b c],

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[b] Abdelhafid et al., *Recent Developments in Math., Statistical and Computational Sciences* (2021).

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Pressure and velocity: impact of the posture

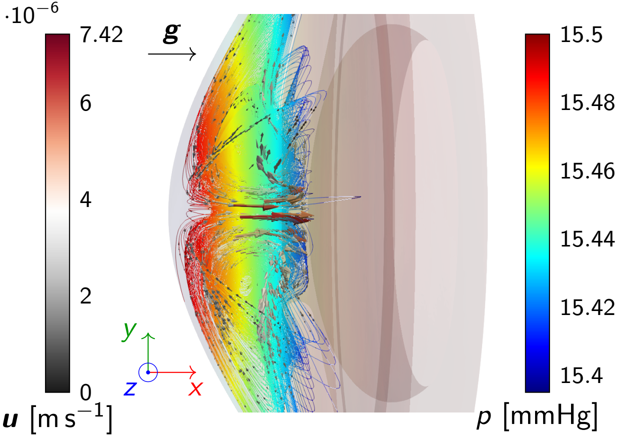

Figure 5: Supine position.

- **Recirculation** of the AH,
- Formation of a **Krukenberg's spindle**, in good agreement with clinical observations and previous studies[abc],
- Fluid dynamics is **strongly influenced by the position of the patient**.

[a] Wang et al., *BioMedical Engineering OnLine* (2016).
[b] Abdelhafid et al., *Recent Developments in Math., Statistical and Computational Sciences* (2021).
[c] Murgoitio-Esandi et al., Translational Vision Science & Technology (2023).

Conclusion and perspectives

- **Coupled fluid-dynamics-heat transfer model** for the AH flow in the human eye,
- Formation of **Krukenberg's spindle**,
- Captured the **importance of the position of the patient** on the AH flow and temperature distribution.

[6] Thomas Saigre et al. "Model order reduction and sensitivity analysis for complex heat transfer simulations inside the human eyeball". Minor revision in IJNMBE. Dec. 2023.

Conclusion and perspectives

- 👍 **Coupled fluid-dynamics-heat transfer model** for the AH flow in the human eye,
- 👍 Formation of **Krukenberg's spindle**,
- 👍 Captured the **importance of the position of the patient** on the AH flow and temperature distribution.

- ⚠ **Computational cost** of the simulations, too high for real-time applications.
- ▶ **Mathematical perspective:** parametrical sensisivity analysis by using ROM[6].
- ▶ **Clinical perspective:** assess the topical administration of drugs.

[6] Thomas Saigre et al. "Model order reduction and sensitivity analysis for complex heat transfer simulations inside the human eyeball". Minor revision in IJNMBE. Dec. 2023.

Thank you for your attention !

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